Semi-Implicit Method for All Mach Number Flow for the Euler Equations of Gas Dynamics on Staggered Grid

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An original numerical method to solve the all-Mach number flow for the Euler equations of gas dynamics on staggered grid is presented. The system is discretized to second order in space on staggered grid, in a fashion similar to the Nessyahu-Tadmor central scheme for 1D model [4] and Jang-Tadmor central scheme for 2D model [5], thus simplifying the flux computation. This approach turns out to be extremely simple, since it requires no equation splitting. We consider the isentropic case and the general case. For simplicity we assume a γ-law gas in both cases.
Both approaches are based on IMEX strategy, in which some term is treated explicitly, while other terms are treated implicitly, thus avoiding the classical CFL restriction due to acoustic waves.
By rescaling the variables the (possibly small) Mach number ε appears in the equations.

1. Isentropic Euler Case:

$$\begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{u}) & = 0 \\ (\rho \mathbf{u})_t + \nabla \cdot (\rho u^2 + p/\varepsilon^2) & = 0 \end{cases} \tag{1}$$

   completed with the relation $p = k\rho^\gamma$. The core if the implicit term contains a non-linear elliptic equation for the pressure, which has to be treated by a fully implicit technique. Because of the non-linearity, it is necessary to adopt an iterative method to compute the pressure. In our numerical experiments Newton's method worked with few iterations.

2. General Euler Case:

$$\begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{u}) & = 0 \\ (\rho \mathbf{u})_t + \nabla \cdot (\rho u^2 + p/\varepsilon^2) & = 0 \\ E_t + \nabla \cdot ((E + p)\mathbf{u}) & = 0 \end{cases} \tag{2}$$

   The system is closed by the (suitably scaled) equation of state $E = \rho\varepsilon^2 \mathbf{u}^2/2 + p/(\gamma - 1)$.

   In this case the implicit term is treated in a semi-implicit fashion, thus avoiding the use of Newton's iterations.

In both cases the schemes are implemented to second order accuracy in time. Suitably *well-prepared* initial conditions are considered, which depend on the Mach number ε. In one space dimension we obtain the same profiles found in the literature ([1],[3] for the isentropic case and [1], [2] for the general Euler system) for all Mach numbers.
Current work is related on the development of second order accurate schemes for 2D problems and higher order accurate schemes for 1D and 2D problems.

References

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